KEY

3. a. $(\sqrt{8} + \sqrt{3})(\sqrt{15} + \sqrt{2})$
$= \underline{\underline{2\sqrt{30} + 4 + 3\sqrt{5} + \sqrt{6}}}$

b. $(\sqrt{3} - \sqrt{5})^2$
$= 3 - 2\sqrt{15} + 5$
$= \underline{\underline{8 - 2\sqrt{15}}}$

c. $(\sqrt{y} + 6)(\sqrt{y} - 6)$
$= \underline{\underline{y - 36}}$

d. $(3x + 11)(2x - 7)$
$= \underline{\underline{6x^2 + x - 77}}$

e. $(x + 3)(x^2 - 4x - 5)$
$= \underline{\underline{x^3 - x^2 - 17x - 15}}$

4. a. $a^5 + b^5$
$= \underline{\underline{(a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)}}$

b. $5x^2 + 27x + 36$
$= \underline{\underline{(5x + 12)(x + 3)}}$

c. $x^3 + 2x^2 - 11x - 12$
$= (x + 1)(x^2 + x - 12)$
$= \underline{\underline{(x + 1)(x - 3)(x + 4)}}$

d. $64p^2 - 36$
$= 4(16p^2 - 9) = \underline{\underline{4(4p + 3)(4p - 3)}}$

5. a. $\dfrac{1}{x^2 + 4x + 3} + \dfrac{1}{x^2 - 1}$
$= \dfrac{1}{(x + 1)(x + 3)} \cdot \dfrac{x - 1}{x - 1}$
$+ \dfrac{1}{(x + 1)(x - 1)} \cdot \dfrac{x + 3}{x + 3}$
$= \dfrac{x - 1 + x + 3}{(x + 1)(x + 3)(x - 1)}$
$= \dfrac{2(x + 1)}{(x + 1)(x + 3)(x - 1)}$
$= \underline{\underline{\dfrac{2}{x^2 + 2x - 3}}}$

b. $\dfrac{9 - x^2}{3x + 9} \div \dfrac{x^2 - 5x + 6}{x^2 + 2x + 1}$
$= \dfrac{(3 + x)(3 - x)}{3(x + 3)} \cdot \dfrac{(x + 1)^2}{(x - 2)(x - 3)}$
$= \underline{\underline{-\dfrac{(x + 1)^2}{3(x - 2)}}}$ or $\underline{\underline{-\dfrac{x^2 + 2x + 1}{3x - 6}}}$

6. a. $5x^{-\frac{2}{3}} \cdot 7x^{\frac{3}{4}} = \underline{\underline{35x^{\frac{7}{20}}}}$

b. $\sqrt{125} \div \sqrt[3]{25}$
$= 5^{\frac{3}{2}} \div 5^{\frac{2}{3}}$
$= \underline{\underline{5^{\frac{5}{6}}}}$ or $\underline{\underline{\sqrt[6]{3125}}}$

c. $\dfrac{12}{\sqrt{19} + 4} \cdot \dfrac{\sqrt{19} - 4}{\sqrt{19} - 4}$
$= \dfrac{12(\sqrt{19} - 4)}{19 - 16}$
$= \underline{\underline{4\sqrt{19} - 16}}$

d. $\dfrac{24}{\sqrt[3]{2}} \cdot \dfrac{\sqrt[3]{4}}{\sqrt[3]{4}}$
$= \dfrac{24\sqrt[3]{4}}{2} = \underline{\underline{12\sqrt[3]{4}}}$

e. $\sqrt[6]{36} = (6^2)^{\frac{1}{6}} = 6^{\frac{1}{3}} = \underline{\underline{\sqrt[3]{6}}}$

f. $7\sqrt{45} + \dfrac{10}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}}$
$= 21\sqrt{5} + 2\sqrt{5} = \underline{\underline{23\sqrt{5}}}$

g. $\log_9 27 = x$
$9^x = 27$
$3^{2x} = 3^3$
$x = \dfrac{3}{2}$
$\underline{\underline{\quad}}$

h. $\log_7 30 + 2\log_7 5$
$= \log_7(30 \times 5^2)$
$= \underline{\underline{\log_7 750}}$

i. $\left(\dfrac{5}{x - 2} + 1\right) \div \left(\dfrac{5}{x + 3} - 1\right)$
$= \dfrac{5 + x - 2}{x - 2} \cdot \dfrac{x + 3}{5 - x - 3}$
$= \dfrac{(x + 3)(x + 3)}{(x - 2)(2 - x)}$
$= \underline{\underline{-\dfrac{(x + 3)^2}{(x - 2)^2}}}$ (*Note:* No canceling is possible!)

7. a. $3^{2x} = 457$
$2x \cdot \log 3 = \log 457$
$x = \dfrac{\log 457}{2 \cdot \log 3}$
$x = 2.78746\ldots$
$S = \{2.78746\ldots\}$

b. $2 + \sqrt{x - 2} = x$
$\sqrt{x - 2} = x - 2$
$x - 2 = x^2 - 4x + 4$
$0 = x^2 - 5x + 6$
$0 = (x - 2)(x - 3)$
$x = 2$ or $x = 3$
check: $2 + \sqrt{2 - 2} = 2$ ✓
check: $2 + \sqrt{3 - 2} = 3$ ✓
$S = \{2, 3\}$

c. $x + \dfrac{1}{x - 2} = \dfrac{x - 3}{2 - x}, x \neq 2$
$x^2 - 2x + 1 = -x + 3$
$x^2 - x - 2 = 0$
$(x - 2)(x + 1) = 0$
~~$x = 2$~~ ext. or $x = -1$
$S = \{-1\}$

8. $f(x) = \dfrac{x^3 + 2x^2 - 11x - 12}{x^2 - x - 6}$
(*Note:* Numerator is the same as in Problem 4c.)

1. $g(x) = 3^x$ is exponential since it has a variable in the exponent.

2. $-7^4 = \underline{\underline{-2401}}$

3. $5 \cdot 8 + 2 \cdot 4^3$
$= 40 + 128$
$= \underline{\underline{168}}$

4. a. $x = \log_b y$ if and only if $y = b^x$.
b. $\frac{x^a}{x^b} = x^{a-b}$
c. $\log xy = \log x + \log y$
d. $x^{\frac{1}{n}} = \sqrt[n]{x}$

5. $(3r^5 m^7)^4 (64r^0 m^{12})^{\frac{2}{3}}$
$= 81r^{20}m^{28} \cdot 16m^8$
$= \underline{\underline{1296r^{20}m^{36}}}$

6. $\sqrt[9]{32586.4} = \underline{\underline{3.172842...}}$

7. $\frac{3.7495 \times 10^{-21}}{7.24 \times 10^{17}}$
$= 5.178867... \times 10^{-39}$
$\approx \underline{\underline{5.18 \times 10^{-39}}}$

8. $f(x) = 7^x$
$x = 7^{f^{-1}(x)}$
$\underline{\underline{f^{-1}(x) = \log_7 x}}$

9. $\log_{25} x = \frac{3}{2}$
$25^{\frac{3}{2}} = x$
$\underline{\underline{125 = x}}$

10. Let $x = \log_6 37.9$
$\therefore 6^x = 37.9$
$x = \frac{\log 37.9}{\log 6}$
$x = \underline{\underline{2.0287...}}$

11. $\log_x 64 = \frac{2}{3}$
$x^{\frac{2}{3}} = 64$
$x = 64^{\frac{3}{2}}$
$x = \underline{\underline{512}}$

12. $3 \log_5 12 - \log_5 36$
$= \log_5 \frac{12^3}{36}$
$= \underline{\underline{\log_5 48}}$

1. $12a^3b^5 + 18ac = \underline{\underline{6a(2a^2b^5 + 3c)}}$

2. $9x^2 - 36 = 9(x^2 - 4)$
$= \underline{\underline{9(x + 2)(x - 2)}}$

3. $x^2 + 14xy - 15y^2 = \underline{\underline{(x + 15y)(x - y)}}$

4. $4x^2 + 2x - 30 = 2(2x^2 + x - 15)$
$= \underline{\underline{2(2x - 5)(x + 3)}}$

5. $4z^2 + 25z + 6 = \underline{\underline{(4z + 1)(z + 6)}}$

6. $9x^2 - 48x + 64 = \underline{\underline{(3x - 8)^2}}$

7. $x^3 + c^3 = \underline{\underline{(x + c)(x^2 - xc + c^2)}}$

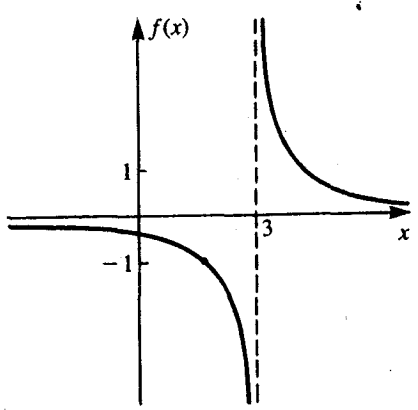
8. $8p^3 - f^3 = \underline{\underline{(2p - f)(4p^2 + 2pf + f^2)}}$

9. $(x + a)y + (x + a)(3p - 4)$
$= \underline{\underline{(x + a)(y + 3p - 4)}}$

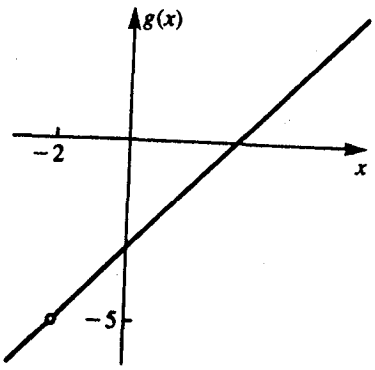
15.
$$x - 2 \overline{)\, 4x^3 - 3x^2 + 7x - 19}$$
$$\underline{4x^3 - 8x^2}$$
$$5x^2 + 7x$$
$$\underline{5x^2 - 10x}$$
$$17x - 19$$
$$\underline{17x - 34}$$
$$15$$
$\therefore \frac{4x^3 - 3x^2 + 7x - 19}{x - 2} = \underline{\underline{4x^2 + 5x + 17 + \frac{15}{x - 2}}}$

16.
$$x + 3 \overline{)\, x^3 + 0x^2 + 0x - 11}$$
$$\underline{x^3 + 3x^2}$$
$$-3x^2 + 0x$$
$$\underline{-3x^2 - 9x}$$
$$9x - 11$$
$$\underline{9x + 27}$$
$$-38$$
$\therefore \frac{x^3 - 11}{x + 3} = \underline{\underline{x^2 - 3x + 9 - \frac{38}{x + 3}}}$

17. $f(x) = \frac{1}{x - 3}$



18. $g(x) = \frac{(x + 2)(x - 3)}{x + 2}$
$g(x) = x - 3$, provided $x \neq -2$.



1. $\dfrac{(x+3)(x-5)}{(x-4)(x-1)} \div \dfrac{(x+3)(5-x)}{(x+4)(x+7)}$

$= \dfrac{(x+3)(x-5)(x+4)(x+7)}{(x-4)(x-1)(x+3)(5-x)}$

$= \underline{\underline{-\dfrac{(x+4)(x+7)}{(x-4)(x-1)}}}$ or $\underline{\underline{-\dfrac{x^2+11x+28}{x^2-5x+4}}}$

2. $\dfrac{x}{x-2} - \dfrac{3}{x+4}$

$= \dfrac{x^2+4x-3x+6}{(x-2)(x+4)}$ $= \underline{\underline{\dfrac{x^2+x+6}{x^2+2x-8}}}$

3. $\dfrac{x^5+y^5}{x+y} = \dfrac{(x+y)(x^4-x^3y+x^2y^2-xy^3+y^4)}{x+y}$

$= \underline{\underline{x^4-x^3y+x^2y^2-xy^3+y^4}}$

4. $\dfrac{x+\dfrac{2}{x+3}}{2+\dfrac{x}{x+3}} \cdot \dfrac{x+3}{x+3}$

$= \dfrac{x^2+3x+2}{2x+6+x}$

$= \dfrac{(x+1)(x+2)}{3(x+2)}$

$= \underline{\underline{\dfrac{x+1}{3}}}$

5. $(3x-7)(x^2-4x+5)$

$= \underline{\underline{3x^3-19x^2+43x-35}}$

6. x^3+125

$= \underline{\underline{(x+5)(x^2-5x+25)}}$

7. $64x^2-16 = 16(4x^2-1)$

$= \underline{\underline{16(2x+1)(2x-1)}}$

8. $P(x) = x^3-5x^2-8x+48$

a. $P(-3) = -27-45+24+48 = 0$, Q.E.D.

b. $(x+3)$ is a factor. By synthetic substitution:

-3	1	-5	-8	48
		-3	24	-48
	1	-8	16	0

$P(x) = (x+3)(x^2-8x+16)$

$\underline{\underline{P(x) = (x+3)(x-4)^2}}$

9.
$$\begin{array}{r} 3x^2+4x-5 \\ x-6\overline{)\,3x^3-14x^2-29x+5} \\ \underline{3x^3-18x^2} \\ 4x^2-29x \\ \underline{4x^2-24x} \\ -5x+5 \\ \underline{-5x+30} \\ -25 \end{array}$$

$\therefore \dfrac{3x^3-14x^2-29x+5}{x-6} = \underline{\underline{3x^2+4x-5-\dfrac{25}{x-6}}}$

10. $x+\dfrac{1}{x-1} = \dfrac{x}{x-1}, x \neq 1$

$x^2-x+1 = x$

$x^2-2x+1 = 0$

$(x-1)^2 = 0$

~~$x = 1$~~ Extraneous

$S = \emptyset$

1. $\sqrt{18}+2\sqrt{50}-\sqrt{98}$

$= 3\sqrt{2}+10\sqrt{2}-7\sqrt{2}$

$= \underline{\underline{6\sqrt{2}}}$

2. $\dfrac{21}{\sqrt{12}+\sqrt{5}} \cdot \dfrac{\sqrt{12}-\sqrt{5}}{\sqrt{12}-\sqrt{5}}$

$= \dfrac{21(\sqrt{12}-\sqrt{5})}{12-5}$

$= 3(2\sqrt{3}-\sqrt{5})$

$= \underline{\underline{6\sqrt{3}-3\sqrt{5}}}$

3. $\dfrac{1-\dfrac{1}{\sqrt{3}}}{1+\dfrac{1}{\sqrt{3}}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{\sqrt{3}-1}{\sqrt{3}+1} \cdot \dfrac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \dfrac{3-2\sqrt{3}+1}{3-1}$

$= \underline{\underline{2-\sqrt{3}}}$

4. $\sqrt{x+3}+\sqrt{x-2} = 5$

$\sqrt{x+3} = 5-\sqrt{x-2}$

$x+3 = 25-10\sqrt{x-2}+x-2$

$10\sqrt{x-2} = 20$

$x-2 = 4$

$x = 6$

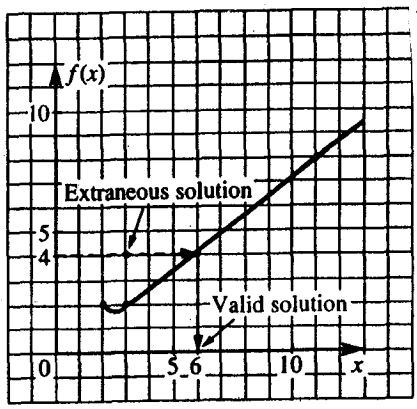
Check: $\sqrt{6+3}+\sqrt{6-2} = 5$ ✓

$S = \{6\}$

7. $f(x) = x-\sqrt{x-2}$

x	$f(x)$
11	8
6	4
3	2
2.5	1.792...
2	2
0	no real value

8.



9. Domain $= \{x: x \geq 2\}$

10. $f(x) = 0$:

$0 = x-\sqrt{x-2}$

$\sqrt{x-2} = x$

$x-2 = x^2$

$0 = x^2-x+2$

$b^2-4ac = 1-4 \cdot 1 \cdot 2 = -7$

$\therefore$ There are no real values of x.

11. $f(x) = 4$:

$4 = x-\sqrt{x-2}$

$\sqrt{x-2} = x-4$

$x-2 = x^2-8x+16$

$0 = x^2-9x+18$

$(x-3)(x-6) = 0$

$x = 3$ or $x = 6$

Checks:

$4 \stackrel{?}{=} 3-\sqrt{1}$ $\quad 4 \neq 2$

$4 \stackrel{?}{=} 6-\sqrt{4}$ $\quad 4 = 4$ ✓

$x = 3$ is extraneous. $x = 6$ is valid.

1. a. $13+19+\ldots$; $d = 19-13 = 6$

$\therefore t_{27} = 13+26(6)$

$= \underline{\underline{169}}$

b. $S_{27} = \dfrac{27}{2}(13+169)$

$= \underline{\underline{2457}}$

c. $445 = 13+(n-1)(6)$

$\dfrac{445-13}{6} = n-1$

$\underline{\underline{73 = n}}$

2. a. $50+48+\ldots$; $r = \dfrac{48}{50} = 0.96$

$867.373 = 50 \cdot \dfrac{1-0.96^n}{1-0.96}$

$0.96^n = 0.306101\ldots$

$n = \dfrac{\log 0.306101\ldots}{\log 0.96}$

$\underline{\underline{n = 29}}$

b. $S = \dfrac{50}{1-0.96}$

$= \underline{\underline{1250}}$

3. $35, \underset{①}{\_\_}, \underset{②}{\_\_}, \underset{③}{\_\_}, \underset{④}{\_\_}, \underset{⑤}{\_\_}, \underset{⑥}{98}$

$98 = 35+(6-1)(d)$

$12.6 = d$

$35, \underline{47.6}, \underline{60.2}, \underline{72.8}, \underline{85.4}, 98$

$\sum_{k=1}^{5} 2^k+3 = 5+7+11+19+35$

$= \underline{\underline{77}}$

b. $7-5 = 2$ and $11-7 = 4$, *not* arithmetic

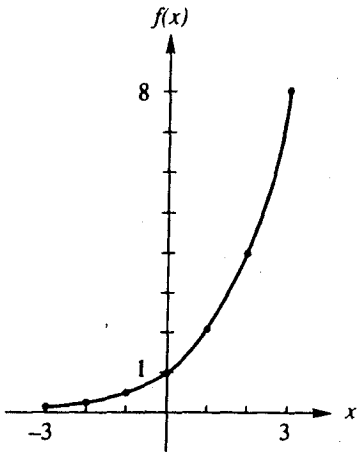
$7 \div 5 \neq 11 \div 7$, not geometric

$\underline{\underline{\text{neither}}}$

1. $y = a \cdot b^x$

2. $f(x) = 2^x$

x	$f(x)$
-3	1/8
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8



3. $f(x) = 3 \cdot 5^x$

$f(3) = 3 \cdot 5^3 = \underline{\underline{375}}$

$f(-2) = 3 \cdot 5^{-2} = \underline{\underline{\frac{3}{25}}}$

$f(0) = 3 \cdot 5^0 = \underline{\underline{3}}$

4. $(ab)^k = \underline{\underline{a^k b^k}}$

5. $\frac{x^a}{x^b} = \underline{\underline{x^{a-b}}}$

6. $(y^u)^p = \underline{\underline{y^{up}}}$

7. y, exponent
x, base
x^y, power

8. $2^{3.7} = \underline{\underline{12.99603\ldots}}$

9. $0.8^{-\frac{3}{7}} = \underline{\underline{1.100355\ldots}}$

10. 2^3 is 8 and 2^4 is 16.
So $2^{3.7}$ is between 8 and 16, which the answer to Problem 8 is.

11. $8^{-\frac{2}{3}} = \frac{1}{8^{2/3}} = \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{2^2} = \frac{1}{4} = 0.25$, Q.E.D.

12. $5^x = 625$, so $\underline{\underline{x = 4}}$.

13. $100^\circ = \underline{\underline{1}}$.

14. $(4x^6y^{10})^3 = \underline{\underline{64x^{18}y^{30}}}$

15. $\sqrt[6]{(11^{7.5})(11^{-5.1})} = (11^{2.4})^{\frac{1}{6}}$
$= 11^{0.4}$

16. $(6x^{\frac{2}{3}})(5x^{-\frac{1}{5}}y^7) = \underline{\underline{30x^{\frac{7}{15}}y^7}}$